OAC 2025

OAC Problem Writing Committee

21 September 2025

Round 1 Instructions

1. The duration of the exam is **75 minutes** and it consists of **45 questions** (all MCQs).
2. The exam will be available during the following **6-hour window**:
   - **IST (Indian Standard Time):** 18:00 – 00:00, 21 Sept 2025
   - **UTC (Coordinated Universal Time):** 12:30 – 18:30, 21 Sept 2025
   - **US Eastern Time (EDT):** 08:30 – 14:30, 21 Sept 2025
   - **US Pacific Time (PDT):** 05:30 – 11:30, 21 Sept 2025

   You may begin at any time within this window, but once started, you will only have 75 minutes.
3. The exam format is **open book** – you may use any **offline resources** (books, notes, calculators, etc.).
4. The use of the **internet, AI tools, or communication with others** is strictly prohibited.
5. This is an **individual contest** – collaboration or answer-sharing will result in disqualification.
6. The **marking scheme** is:
   - Correct answer: **+5**
   - Wrong answer: **0**
   - Unattempted: **+1**
   - Maximum score: **225**
7. The **Top 15-20 participants** will qualify for Round 2, to be held on 4 October 2025. The results of the first round will be declared by **30 September 2025**
8. By starting the exam, you agree to follow all the rules in good faith.

Good luck! Aim for the stars.

Questions

1. Xinxin the astronomer is monitoring a nearby star system continuously for longer than the planet's orbital period. The system contains a planet on a circular orbit ($e = 0$) with orbital radius $a = 0.8\,\mathrm{AU}$. The host star has bolometric luminosity $L_\star = 1.00\,L_\odot$ and effective temperature $T_{\mathrm{eff}} = 5500\,\mathrm{K}$. What is the probability that Xinxin's line of sight will allow her to observe the planet transit its star?

   A. 0.91%
   B. 0.64%
   C. 0.47%
   D. 0.11%

2. A black hole is located at the center of an accretion disk. A star is orbiting the black hole with the orbital plane perpendicular to the plane of the disk and to the line of sight. The argument of perihelion is $\omega = 270^\circ$ and the eccentricity is $e = 0.3$. The period of the orbit is $T = 200\,\mathrm{years}$. When the star hits the disk, a bright flash of light is emitted. What is the shortest time interval between two consecutive flashes?

   A. 75.8 years
   B. 62.4 years
   C. 94.2 years
   D. 58.1 years

3. A peculiar individual, Roderick, sees that there are two enormous balls under gravitational influence with each other and are isolated from other external forces. One ball is twice as massive as the other. They follow circular orbits around each other with a period of T. If, for some reason, the balls stopped in their orbits, how long will it take for the balls to touch (i.e. time for them to collide), assuming the balls, though enormous, are much farther away than their respective radii?

   A. $\frac{T}{\sqrt{72}}$
   B. $\frac{T}{\sqrt{16}}$
   C. $\frac{T}{\sqrt{64}}$
   D. $\frac{T}{\sqrt{32}}$

4. Consider a star having a number density profile as $n(r) = n_0\, e^{-\alpha r^4}$ where r is the distance from the center of the star and n_0 is the number density at the center. The total number of molecules in the star is proportional to?

   A. $n_0\,\alpha^{-3/4}$
   B. $n_0\,\alpha^{-3}$
   C. $n_0\,\alpha^{1/4}$
   D. $\sqrt{n_0}\,\alpha^{1/2}$

5. An observant observer observes the moon over the course of a month. Say the eccentricity of the moon is the only cause for the observant observer to see more than half of the moon throughout the observation. If $e = 0.0549$ for the moon, what fraction of it is visible over the month?

   A. 50.1%
   B. 53.5%
   C. 59.2%
   D. 62.5%

6. Teo had extraordinary abilities, one of which was the ability to talk to planets. One day, he was arguing with a planet and got so frustrated, he scolded the planet. The planet, being the innocent lump of matter it was, felt really bad and wanted to hug Teo. It tried to reach out towards him, but could only slightly do so, since it was bound by the gravity of its host star. Assume the planet is perturbed only slightly from its position so that its new position is $r = R + x$, where $x \ll R$ and R is the original radius of its circular orbit. The mass of the star is M. What is the time period of the simple harmonic motion of the planet about its mean position, R?

A. $2\pi\sqrt{\frac{R^3}{GM}}$

B. $2\pi\sqrt{\frac{2R^3}{GM}}$

C. $2\pi\sqrt{\frac{2R^3}{3GM}}$

D. $2\pi\sqrt{\frac{R^3}{3GM}}$

7. A rocket scientist decides to launch a rocket to the antipode from its launch site, with ε being the eccentricity of the orbit. In which angle should the rocket fire relative to the ground to achieve this? Assume the Earth is a spherically symmetric sphere, and ignore all other motions such as the Earth's rotation and revolution.

   A. $\frac{1}{\varepsilon}$

   B. $\tan^{-1}\frac{1}{\varepsilon}$

   C. $\sin^{-1}\frac{1}{\varepsilon}$

   D. $\tan^{-1}\varepsilon$

8. The redshift of the Carina Dwarf Galaxy (a Milky Way satellite) is $z = 0.00077$. The distance to the galaxy is

   A. 3.3 Mpc

   B. 0.1 Mpc

   C. 1.1 Mpc

   D. 20 Mpc

9. The Roche lobe is the region around a star in a binary system within which orbiting material is gravitationally bound to that star. It is an approximately teardrop-shaped region bounded by a critical gravitational potential, with the apex of the teardrop being at the L_1 lagrange point. Consider a binary system with both stars of equal mass M separated by distance a in circular orbits about the common center of mass. Find the gravitational potential of the Roche lobe boundary in a frame co-rotating with the stars.

   A. $-\frac{4GM}{a}$

   B. $-\frac{2GM}{a}$

   C. $-\frac{GM}{a}$

   D. $-\frac{GM}{2a}$

10. An observer on the Earth's surface at a latitude of 50° observes a satellite which orbits the Earth in a circular orbit inclined at an angle of 12° with the equatorial plane and with a height H above the earth's surface. If it is above the horizon for one-fourth of its time period, calculate the height H. Neglect the earth's rotation.

    A. 4467 km

    B. 8592 km

    C. 6783 km

    D. 5068 km

11. Daniel, a strange guy, finds a strange force as well. He finds it to be a central force proportional to r^n, where r is the distance from the source of the force to the body affected by it (perhaps, think of it as a strange type of "gravity"). What value of n could make it such that closed, stable, nearly circular orbits can occur by bodies under the influence of this force (that is, orbits without precession each period)?

    A. 10

    B. 11

    C. 12

    D. 13

12. Equatorial coordinates of Vega are $\alpha = 18^h37^m$ and $\delta = 39\,N$ and that of Deneb are $\alpha = 20^h43^m$ and $\delta = 45\,N$. Now, suppose the earth changed its orientation and started spinning about Vega rather than Polaris. At a particular time, the altitude of Deneb is 20° at the location where Deneb just becomes circumpolar. What is the hour angle of Deneb at this time?

    A. 6.3^h

    B. 4.5^h

    C. 7.1^h

    D. 6.7^h

13. A spectroscopic binary system shows maximum radial velocities $K_1 = 30\,\mathrm{km\,s^{-1}}$ and $K_2 = 10\,\mathrm{km\,s^{-1}}$. The orbital period is $P = 30\,\mathrm{days}$. Assuming a circular, edge-on orbit, let the masses of the two stars be M_1 and M_2. Which of the following options gives the corect value of (M_1, M_2)?

    A. $(0.05\,M_\odot, 0.15\,M_\odot)$

    B. $(0.5\,M_\odot, 1.5\,M_\odot)$

    C. $(0.05\,M_\odot, 0.05\,M_\odot)$

    D. $(1.0\,M_\odot, 1.0\,M_\odot)$

14. The first acoustic peak in the CMB corresponds to the angular size of the sound horizon at recombination, about 1°. If the physical scale of the sound horizon is 150 Mpc, what is the comoving distance to the last scattering surface implied by this observation?

    A. 1500 Mpc

    B. 5500 Mpc

    C. 8500 Mpc

    D. 10000 Mpc

15. A planet named Jeff orbits the star Tiger in an elliptical orbit having eccentricity $e = 0.56$ and time period of revolution $P = 2000$ Jeff days. The planet's equatorial plane is tilted to its plane of orbit by an angle of 19°. Jeff is closest to Tiger at its ascending node. How much time after Jeff's passage through the ascending node will Tiger achieve half of its minimum declination for the first time?

    A. 39 Jeff days

    B. 40 Jeff days

    C. 41 Jeff days

    D. 42 Jeff days

16. Which of the following features of the moon is the last to enter the umbral region during the total lunar eclipse of September 7, 2025?

    A. Copernicus crater

    B. Mare Crisium

    C. Mare Fecunditatis

    D. Langrenus crater

17. DuNL1, a primordial black hole, was left alone in the Milky Way after the death of all his family members. Being near his end too, he decides to pay our Solar System a visit before leaving this universe forever. Astronomers at Earth prepared to welcome him by throwing a big rock at him (so that it could live longer) but sadly they missed DuNL1 and the rock ended up on a trajectory to Vega. Just as DuNL1 was about to vanish, astronomers observed something interesting- the amount of radiation emitted by DuNL1 in his last t ns before death was equal to the radiation emitted by the Sun in the same t ns. What was the value of t recorded by the astronomers?

    A. 260

    B. 390

    C. 0.4

D. 1920

18. The rock thrown into space missed DuNL1, the primordial black hole, and is now on a straight path to Vega, moving with $0.98c$. Vega is often used as a reference star for calibrating magnitudes, and its apparent magnitude can be taken to be zero. The absolute magnitude of Vega is around 0.6^m. Including the effects of interstellar extinction, estimate the time it would take for the rock to reach Vega.

    A. 25 years

    B. 30 years

    C. 100 years

    D. 420 years

19. Consider a flat cosmological constant dominated universe, with the value of the cosmological constant being on the order of $10^{-40}s^{-2}$. For light from an object to be seen today, what could the object's maximum proper distance be at the time when the light was emitted?

    A. 600 Mpc

    B. 5 Gpc

    C. 2 Tpc

    D. 800 kpc

20. During a total lunar eclipse, the Moon passes directly through the center of the Earth's umbral shadow. For a brief period before entering the umbra and after exiting it, the entire Moon is completely within the Earth's penumbral shadow but not touching the umbra. Calculate the time duration for which the entire disc of the Moon is within the penumbral shadow alone during this eclipse. Assume that all orbits are circular and coplanar, all bodies are spherical and the mass of the moon is much lesser than the mass of the Earth.

    A. 100s

    B. 210s

    C. 7040s

    D. 3520s

21. In a planet's apparent motion as seen from Earth, it appears to switch between prograde (eastward) and retrograde (westward) motion against the background stars. The transition points where this motion appears to halt are called stationary points. When Venus is at the stationary point, what is Earth's phase as seen from Venus? Assume coplanar, circular orbits.

    A. 0.94

    B. 0.92

    C. 0.97

    D. 0.89

22. Consider a galaxy with negligible thickness(imagine a thin, circular disk) with area density $\sigma(r)$ at distance r from the center. Consider that a test mass in a keplerian orbit revolves around it in a circular orbit with velocity v, from a distance of r from the galactic center. Which one of the following differential equations holds for this scenario?

    A. $2\pi rG\sigma(r) = v(r)^2 + 2\,v(r)\,r\,\frac{dv(r)}{dr}$

    B. $\pi rG\sigma(r) = v(r)^2$

    C. $2\pi rG\sigma(r) = 2\,r\,v(r)\,\frac{dv(r)}{dr}$

    D. $\pi rG\sigma(r) = v(r)^2 + 2\,v(r)\,\frac{dv(r)}{dr}$

23. In the far future, humanity builds planet-sized rotating toroidal structures for interstellar travel, designed to sustain human life for a very long time. The toroid is solid (i.e not hollow), and has a diameter D and density ρ. The cross-sectional radius R of the toroid is much smaller than the radius of the toroid ($R \ll D/2$). A communication satellite (which is obviously also much smaller than the planet-sized toroid) is orbiting very closely to the edge of the toroid at a distance r from the center of the cross section (the orbit is in a plane perpendicular to the plane of the toroid). The period of the satellite is

A. $\sqrt{\frac{4\pi}{\rho G}}\left(\frac{r}{R}\right)^{3/2}$

B. $\sqrt{\frac{8\pi}{\rho G}}\frac{r}{R}$

C. $\sqrt{\frac{2\pi}{\rho G}}\left(\frac{r}{R}\right)^{3/2}$

D. $\sqrt{\frac{2\pi}{\rho G}}\frac{r}{R}$

24. A satellite is orbiting the Earth in a circular orbit with radius $5R_\oplus$. The inclination of the orbit with respect to the equator is $i = 20°$. We want to change the right ascension of the ascending node by 30°. What is the required Δv for this maneuver?

    A. 1.7 km/s

    B. 2.1 km/s

    C. 0.9 km/s

    D. 1.1 km/s

25. A supernova at redshift $z = 0.05$ has observed peak apparent magnitude $m = 17.0$. What is the approximate absolute magnitude M of the supernova?

    A. -19.65

    B. -19.76

    C. -19.5

    D. -20

26. Camila was really bored, and missed her hometown. She planned to leave on her super fancy jet. She lived in East Atlanta ($34° N, 84° W$), and half of her heart was in her hometown, Havana ($23° N, 82° W$). While on the flight, she was really nostalgic and overcome with emotions, and didn't realize she had actually overshot Havana. The nearest airport on her line of travel (she travelled along the shortest path between Havana and East Atlanta) lay on the equator. What is the longitude of the airport? Assume that she always stayed in the Northern hemisphere (except obviously when at the Equator).

    A. $79.25° W$

    B. $80.67° W$

    C. $78.61° W$

    D. $56.51° W$

27. A spherical dust particle (can be treated as an ideal blackbody) having radius r orbits the Sun in a circular orbit of radius $a > 1$ AU. In the frame of the dust particle, it experiences a torque due to radiation pressure from the Sun, and slowly spirals inwards. Let the time it takes to fall into the Sun be t_0. Find the dependence of t_0 on a.

    A. $t_0 \propto a$

    B. $t_0 \propto a^2$

    C. $t_0 \propto a^{-1}$

    D. t_0 is independent of a

28. What was the redshift at the time radiation decoupled from matter and the universe became transparent?

    A. 950

    B. 1100

    C. 1300

    D. 1500

29. Consider an interstellar rock barging into the Solar System in a parabolic path. Consider its trajectory to be Keplerian and ignore the effects of the other planets. Find the maximum speed the rock can have to avoid hitting the Sun.

A. 620 km/s

B. 200 km/s

C. 1240 km/s

D. 890 km/s

30. The southern sky is of course the easiest to learn, as well as to find stars in! There is a missing star X brighter than magnitude 3 in the above sky chart. Given that the Galactic Longitudes of Acrux and Alpha Centauri are 300° and 316° respectively, estimate the galactic longitude of the missing star X.

A. 240°

B. 260°

C. 280°

D. 300°

31. KW ($M = 20M_\odot$, $R = 1000R_\odot$) is a suspicious star, thought to be made up of suspicious matter. A mind bending property of these suspicious stars is that their radii are independent of their mass. Teo kw, a Singaporean researcher, decided to study this star for his thesis project. To ease his calculations, he assumed the star to be an ideal spherically symmetric ball of monoatomic ideal gas. He used several models to model the interior of KW, one of which was to assume the interior gas to follow the relation $\ln P = n \ln \rho + k$, where P is the pressure of the gas and ρ is its density. What is the total energy of KW as predicted by Teo kw using this model?

A. -4.55×10^{40} J

B. 4.55×10^{41} J

C. -9.11×10^{40} J

D. -1.14×10^{41} J

32. The apparent brightness of a star as observed from Earth is diminished by interstellar extinction due to dust and gas in the atmosphere and the interstellar medium. The optical depth (τ) is a measure of this absorption and scattering. For this problem, consider the optical depth of the atmosphere along the line of sight to be $\tau = 0.2$. Five stars with known absolute magnitudes are observed from the Earth. You are given the following data for five stars:

Star	m_V	Altitude (°)	M_V
Cirdla	4.5	80	-1.3
Tahska	7.3	82	4.9
Garhsim	3.9	72	0.7
Nuheat	5.5	79	2.5
Tnamus	2.1	85	-3.5

Accounting for atmospheric extinction, what is the average distance of these 5 stars from Earth?

A. 69 pc

B. 35 pc

C. 70 pc

D. 87 pc

33. Ethan watches a star of declination 69° at the University of Toronto's David Dunlap Observatory (latitude 44° N). He finds that the star's greatest azimuth East or West is closest to which of the following?

A. 20°

B. 30°

C. 40°

D. 50°

34. Farazard, having no work to do this weekend, decides to use his magical powers to turn into a (bright?) star. He turns into a star having radius $R_\odot$, and temperature $3000\,K$ (he wants to stay 'cool'). $400R_\odot$ from him, there lies a disc of radius $300R_\odot$, with its surface perpendicular to the line joining its center to Farazard. The disc is a good conductor of heat, and has emissivity 12/17. The surroundings are a perfect vacuum. Let the steady state temperature of the disc be $100\,n^{1/4}$. Find n. Assume the size of the star to be much smaller than other distances.

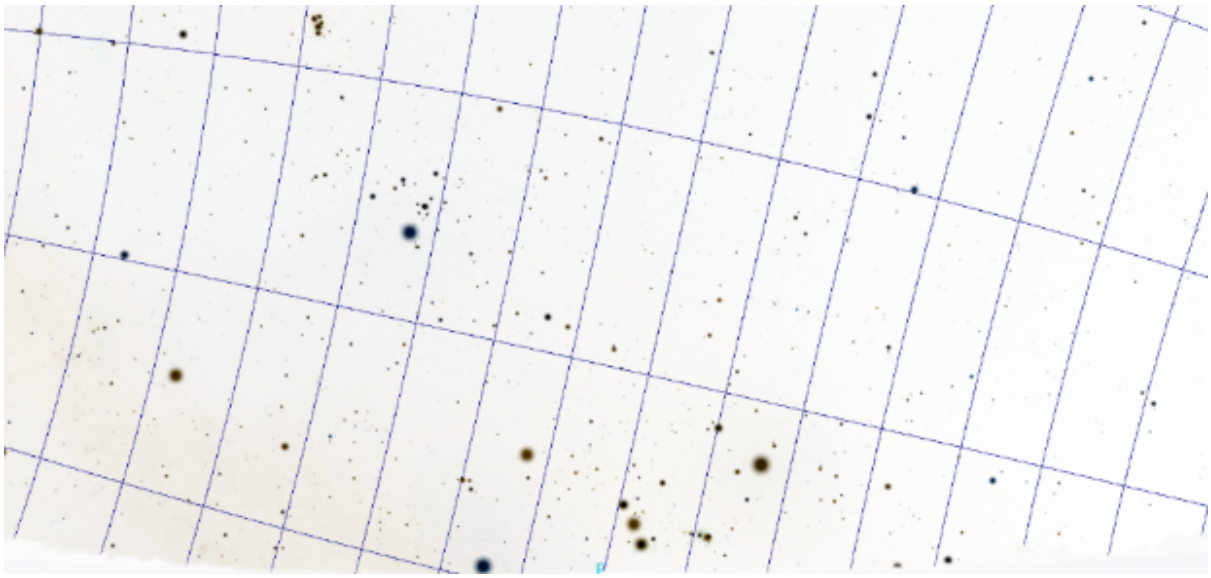
A. 1.8

B. 2.8

C. 3.5

D. 0.7

35. Jassi is an ardent astronomer and stargazer. He likes to have tea watching the stars go about their daily diurnal motion. Given is an image of the Eastern horizon, taken by Jassi when he finished his cup of tea. The equatorial grid is also drawn. You know that the time of observation is around midnight. What is the approximate time of the year this photo has been taken?



A. late September-early October

B. late January-early February

C. late June-early July

D. late December-early January

36. You discover a pulsar of radius R_p. The pulsar is spinning with an extremely high frequency. As you were studying it, a black circular spot of radius $R_s = 0.1\,R_p$ appeared on its equator. What is the change in the magnitude of the pulsar due to this spot? Assume that you can resolve the disc of the pulsar.

    A. 0.149

    B. 0.010

    C. 0.137

    D. 0.201

37. Head of the astrophysical phenomenon department, MishraG, wanted to find out if the Sun really passes through the vernal equinox. To do this, he measured the equatorial coordinates of the Sun at the time when it was precisely at the (people told him it will be) vernal equinox, using his ultra-precise coordmeasurerG device located at the equator. Find the distance of the Sun from the vernal equinox as recorded by MishraG. (Assume the Earth orbits the Sun in a perfectly circular orbit)

    A. 0

    B. $20.79''$

    C. $20.49''$

    D. $41''$

38. As humans finally established contact with aliens, they were overjoyed. Only to realize later that their joy wouldn't last long...

    The aliens had secretly infiltrated within the human population, and had been spying on us. The humans were horrified to find about this, and started praying to god, who they believed resided at the North Celestial Pole. However, they were confused when they saw the sky maps given by astrologers, which indicated the NCP to be at the position shown by the red arrow.

    Roughly how many years old is the astrologers' sky map? Assume the age of the map to be less than earth's precession period.

    A. 2500 years

    B. 4000 years

    C. 23500 years

    D. 22000 years

39. One day, a mysterious signal struck the Earth. The source of the signal has galactic coordinates $l = 12.5°$ and $b = 0°$. The signal seems to come from a star in our galaxy not too far away from us ($d \ll 8\,\mathrm{kpc}$). We measure the radial velocity of the star to be $v_r = 5\,\mathrm{km\,s^{-1}}$. We also measure the

$B - V$ color index to be equal to 1.3. What is the temperature of the star? Take the interstellar extinction to be $A_V = 1^m\,\mathrm{kpc}^{-1}$. The ratio of the total extinction to the color excess is 3.2. Assume the star is a black body so the following relation between the temperature and the intrinsic color index of the star is true:

$$T = 4600\,\mathrm{K}\left(\frac{1}{0.92(B-V)+1.7}+\frac{1}{0.92(B-V)+0.62}\right)$$

A. 7092 K

B. 5326 K

C. 3889 K

D. 4620 K

40. Two planets Bani (semi-major axis 2 AU, eccentricity 0.3) and Pani (semi-major axis 12 AU) orbit the star Mish-Sri in coplanar elliptical orbits. If the maximum difference in the apparent magnitude of Pani as seen from Bani is 4.0, then find the difference in apparent magnitude of Mish-Sri as seen from Pani when it is at its apoastron and as seen from Bani when it is at its periastron.

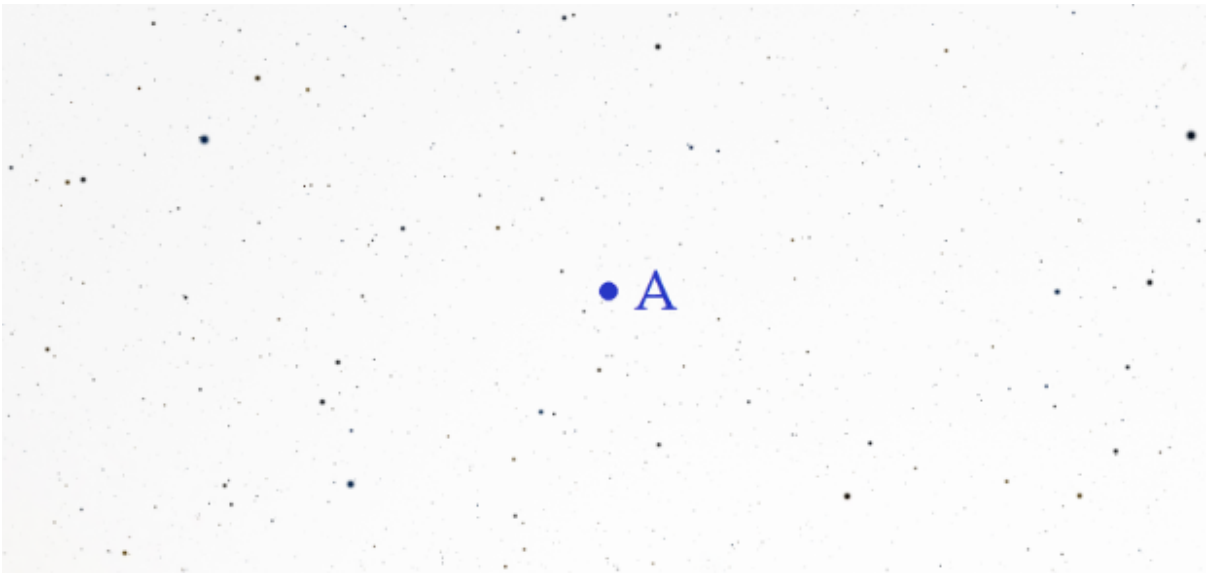
A. 5.56

B. 7.85

C. 3.65

D. 4.84

41. The sky chart for a part of the sky is given above, with a point marked as A. Which of the following Messier objects has the longest great circle arc connecting the point A to it?



A. M95

B. M46

C. M52

D. M80

42. What is the phase of a spherical satellite at the L_4 Lagrange point as seen from the Earth?

A. 0.64

B. 1.0

C. 0.75

D. 0.33

43. Rayan's telescope is located at altitude 1100 m at the Observatoire du Mont-Mégantic (in Quebec, Canada). The diameter of his telescope is 4.20 m with a focal ratio of 6.90. The field of view is $20'$. The CCD pixels in this telescope are squares with an edge length of $20\,\mu$m. The number of megapixels this telescope should have is closest to which of the following?

    A. 50

    B. 60

    C. 70

    D. 80

44. Consider launching a solar panel to space so that it can stay in an (unstable) equilibrium by balancing the force originating from the radiation pressure from the Sun and the gravitational force from the Sun. What would be the area density of the solar panel?

    A. $0.77\,\mathrm{g\,m^{-2}}$

    B. $4.81\,\mathrm{g\,m^{-2}}$

    C. $0.15\,\mathrm{g\,m^{-2}}$

    D. $62.1\,\mathrm{g\,m^{-2}}$

45. Determine the month the following painting, *Starry Night Over the Rhône* by Vincent van Gogh, was drawn. Assume that it was drawn at midnight.

    A. March

    B. June

    C. September

    D. December